

# Joint Force Analysis and Moment Efficiency Index of Cable-Driven Rehabilitation Devices

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**Abstract**—Cable-driven rehabilitation devices (CDRDs) have been studied by many researchers in the past decade. While a CDRD rotates a human joint by generating an assistant moment about the axis of the joint, it also creates a resultant force acting on the joint as long as the assistant moment is nonzero. Such a joint force may cause excessive joint wear or even break the joint if it exceeds a threshold. Thus, it is critical to analyze not only the assistant moment generated by a CDRD to rotate the joint, but also the joint force to have a safe and effective rehabilitation training. This paper studies how a CDRD with three degrees of freedom (DOFs) and four cables exerts a joint force on a general three-DOF human joint. The kinematics and dynamics models of the CDRD are established and the joint force needed to provide the assistant moment is derived mathematically at first. Then, an index to evaluate the efficiency of a CDRD in providing assistant moment (i.e., moment efficiency) is proposed. Lastly, a case study of the flexion and extension of the knee assisted by a CDRD is presented to demonstrate the derivation of the joint force and the usage of the moment efficiency index. The moment efficiency index not only promotes the safety of rehabilitation training, but also provides a guideline for the design of CDRDs.

**Keywords**—joint force, moment efficiency index, cable-driven rehabilitation device, wearable device

## I. INTRODUCTION

Stroke is a comparative cause of permanent disability and severe impairments often come with survivors of stroke [1]. The main symptoms of survivors with impairments are loss of muscle strength, spasticity, and lack of coordination of muscle activation [2]. These lost capabilities are necessary to perform basic activities of daily living. Thus, impairments significantly reduce a survivor's quality of life [3]. Rehabilitation devices with various power-transmission approaches have been proven effective and efficient in helping trainees restore their lost capabilities [4]. Compared with power-transmission approaches such as gear transmission, belt transmission, and ball-screw transmission [5], transmitting power using cables or wires provides a rehabilitation device with many merits, such as low inertia, high payload-to-weight ratio, modularity, and reconfiguration [6], [7]. Therefore, several cable-driven rehabilitation devices (CDRDs) have been proposed in the past

decade [8]–[16]. An example CDRD for the knee and the ankle is shown in Fig. 1.



Fig. 1. An example CDRD for the knee and the ankle

There are usually two categories of interfaces between a CDRD and a trainee, namely, rigid cuff and soft brace. Cable-driven exoskeletons with metal cuffs were developed for upper limb rehabilitation [8], [9] and lower limb rehabilitation [10], respectively. Chen et al. [12] demonstrated a cable-driven exoskeleton with metal cuffs for wrist rehabilitation. A cable-driven robotic device was also proposed in [11] for ankle rehabilitation. Moreover, soft braces were employed in CDRDs for shoulder rehabilitation [13] [14], knee assistance [15], and ankle assistance [16].

To gain the mobility of a human joint, a CDRD usually rotates the joint repeatedly by generating an assistant moment about the axis of the joint [17]. In order to generate such an assistant moment during rehabilitation training, the CDRD needs to exert a force on the joint. Such a force may cause excessive wear of the joint or even break the joint if it exceeds a threshold. Thus, it is critical to analyze not only the assistant moment generated by the CDRD to rotate the joint, but also the joint force exerted by the CDRD on the joint for the sake of safety and the design of CDRDs. Joint force issues have been discussed by a few researchers in the study of human-robot interaction for rehabilitation training. A method of trajectory optimization was proposed for a cable-driven upper arm exoskeleton to minimize the assistant moment and joint force in [18]. A method of evaluating whether a joint force is

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safe was presented for human-care robots in [19]. The threshold of a safe joint force is defined in [19] as the minimal force that can cause injury to a human joint. However, a general analysis of the joint force generated by a CDRD is still not available in the literature. Such a joint force analysis can help evaluate the efficiency of the CDRD in providing assistant moment (called moment efficiency in this paper) with respect to generated joint force.

This paper presents a general analysis of the joint force generated by a CDRD, assuming that there is no collision between cables and the human body, and proposes a moment efficiency index to evaluate the moment efficiency of the CDRD. The rest of the paper is organized as follows. Section II proposes the kinematics and dynamics models of a general three-DOF human joint assisted by a CDRD. In section III, a moment efficiency index is proposed. A joint force analysis of the knee with one degree of freedom (DOF) assisted by a CDRD with four cables is conducted in section IV. Finally, Section V summarizes this paper.

## II. MODELING OF KINEMATICS AND DYNAMICS

In this section, the kinematics and dynamics of a general three-DOF human joint assisted by a three-DOF CDRD with four cables are presented. A three-DOF CDRD with four cables can be fully-constrain [20] and thus, it can assist all three DOFs of the joint.

### A. Kinematics Analysis

The kinematics architecture of a general three-DOF human joint (in aqua) assisted by a three-DOF CDRD (in red) with four cables is shown in Fig. 2.  $A_i$  and  $B_i$  are the two attaching points of the  $i$ th cable on the base and the end-effector, respectively. The base and the end-effector are worn on the two parts of the joint, respectively.  $\mathbf{l}_i \in R^3$  ( $i = 1, 2, 3, 4$ ) is the vector along the  $i$ th cable.  $\mathbf{u}_i$  is the unit vector along the  $i$ th cable. The magnitude of vector  $\mathbf{l}_i$ , represented by scalar  $l_i$ , is the length of the  $i$ th cable between attaching points  $A_i$  and  $B_i$ . The positions of the attaching points  $A_i$  and  $B_i$  are represented by vectors  $\mathbf{a}_i$  and  $\mathbf{b}_i$ , respectively. Obviously,  $\mathbf{a}_i$  is a constant vector in the base frame  $F_b$  and  $\mathbf{b}_i$  is a constant vector in the end-effector frame  $F_e$ . The origin of the base frame is denoted by point  $O$ , and the origin of the end-effector frame  $F_e$  is denoted as point  $P$ . Both  $O$  and  $P$  locate at the rotation center of the end-effector, namely the rotation center of the joint. The attitude of  $F_e$  with respect to  $F_b$  is described by three Euler angles  $\boldsymbol{\Omega} = [\psi, \theta, \phi]^T$  with the Z-Y-X sequence.

Based on the above kinematics notations, the position of the end-effector with respect to the based frame can be described as [21]

$$\mathbf{p} = \mathbf{a}_i - \mathbf{l}_i - \mathbf{b}_i \text{ for } i = 1, 2, 3, 4 \quad (1)$$

from which one has

$$l_i^2 = [\mathbf{a}_i - \mathbf{p} - \mathbf{b}_i]^T [\mathbf{a}_i - \mathbf{p} - \mathbf{b}_i] \text{ for } i = 1, 2, 3, 4 \quad (2)$$

Differentiating (2) with respect to time, and then organizing the four resulting equations into a matrix form, one obtains

$$\dot{\mathbf{l}} = \mathbf{J} \dot{\mathbf{X}} \quad (3)$$

where

$$\dot{\mathbf{l}} = [\dot{l}_1 \ \dot{l}_2 \ \dot{l}_3 \ \dot{l}_4]^T \quad (4)$$

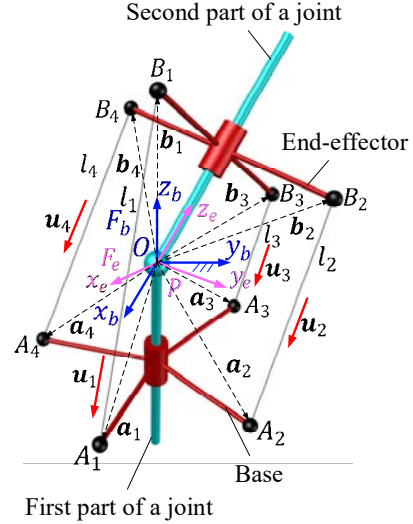


Fig. 2. Kinematics notations of a general three-DOF human joint (in aqua) assisted by a three-DOF four-cable CDRD (in red)

$$\mathbf{J} = [\mathbf{b}_1 \times \mathbf{u}_1 \ \mathbf{b}_2 \times \mathbf{u}_2 \ \mathbf{b}_3 \times \mathbf{u}_3 \ \mathbf{b}_4 \times \mathbf{u}_4]^T \quad (5)$$

$$\dot{\mathbf{X}} = \begin{bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{bmatrix} = [\dot{x} \ \dot{y} \ \dot{z} \ \dot{\theta}_x \ \dot{\theta}_y \ \dot{\theta}_z]^T \quad (6)$$

In the above equations,  $\dot{\mathbf{l}}$  is the velocity vector in the joint space.  $\dot{\mathbf{X}}$  denotes the twist of the end-effector.  $\mathbf{v} = [\dot{x} \ \dot{y} \ \dot{z}]^T$  and  $\boldsymbol{\omega} = [\dot{\theta}_x \ \dot{\theta}_y \ \dot{\theta}_z]^T$  represent the translational and rotational velocity vectors of  $F_b$  with respect to  $F_e$ .  $\mathbf{J}$  is the Jacobian matrix of the CDRD.

The rotation matrix transforming a vector from  $F_e$  to  $F_b$  is [22]–[25]

$$\mathbf{R}_e^b = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \quad (7)$$

where  $C_{angle}$  and  $S_{angle}$  are designated as  $\cos(angle)$  and  $\sin(angle)$ , respectively.  $\mathbf{R}_e^b$  is orthogonal, so the rotation matrix transforming a vector from  $F_b$  to  $F_e$  is [22]–[25]

$$\mathbf{R}_b^e = [\mathbf{R}_e^b]^{-1} = [\mathbf{R}_e^b]^T \quad (8)$$

The rotation matrix transforming the angular velocity of the end-effector from  $F_e$  to  $F_b$  is [22]–[25]

$$\mathbf{R}_\omega^\Omega = \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi / C_\theta & C_\phi / C_\theta \end{bmatrix} \quad (9)$$

where  $T_{angle}$  is designated as  $\tan(angle)$ . To convert angular velocity of the end-effector from  $F_e$  to  $F_b$ , one has

$$\dot{\boldsymbol{\Omega}} = \mathbf{R}_\omega^\Omega \boldsymbol{\omega} \quad (10)$$

where  $\dot{\boldsymbol{\Omega}}$  is the angular velocity of the end-effector in  $F_b$ . The rotation matrix transforming the angular velocity of the end-effector from  $F_b$  to  $F_e$  is [22]–[25]

$$\mathbf{R}_\Omega^\omega = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \quad (11)$$

## B. Dynamics Analysis

Compared to the inertia of the end-effector, the inertia of cables is usually much smaller and can be ignored [26]. Then, the motion of the end-effector can be expressed based on Newton-Euler Formulation with respect to the end-effector frame  $F_e$  as,

$$\mathbf{M}(\mathbf{X})\ddot{\mathbf{X}} + \mathbf{C}(\mathbf{X}, \dot{\mathbf{X}})\dot{\mathbf{X}} + \mathbf{G}(\mathbf{X}) + \mathbf{F}_j(\mathbf{X}) = -\mathbf{J}^T \boldsymbol{\tau} \quad (12)$$

where  $\mathbf{M}(\mathbf{X})$  denotes the mass matrix.  $\mathbf{C}(\mathbf{X}, \dot{\mathbf{X}})$  denotes the Coriolis and centripetal matrix.  $\mathbf{G}(\mathbf{X})$  represents the  $6 \times 1$  gravity vector.  $\mathbf{F}_j(\mathbf{X})$  represents the  $6 \times 1$  joint wrench vector. The mass matrix  $\mathbf{M}(\mathbf{X})$ , Coriolis and centripetal matrix  $\mathbf{C}(\mathbf{X}, \dot{\mathbf{X}})$ , and the gravity matrix  $\mathbf{G}(\mathbf{X})$  in (12) have been defined by several scholars [27], [28]

$$\mathbf{M}(\mathbf{X}) = \begin{bmatrix} m\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_p \end{bmatrix} \quad (13)$$

$$\mathbf{C}(\mathbf{X}, \dot{\mathbf{X}})\dot{\mathbf{X}} = \begin{bmatrix} \boldsymbol{\omega} \times m\mathbf{v}_p \\ \boldsymbol{\omega} \times \mathbf{I}_p \boldsymbol{\omega} \end{bmatrix} \quad (14)$$

$$\mathbf{G}(\mathbf{X}) = \begin{bmatrix} \mathbf{F}_g \\ \mathbf{W}_g \end{bmatrix} \quad (15)$$

$$\mathbf{F}_j(\mathbf{X}) = \begin{bmatrix} \mathbf{F}_j \\ \mathbf{0}_{3 \times 1} \end{bmatrix} \quad (16)$$

where  $\mathbf{v}_p$  is the translational velocity vector of the center of mass of the end-effector in  $F_e$ .  $\mathbf{I}_p$  is the inertia tensor of the end-effector about point  $P$  in  $F_e$ .  $\mathbf{I}_{3 \times 3}$  is the  $3 \times 3$  identity matrix.  $\mathbf{0}_{3 \times 3}$  is the  $3 \times 3$  zero matrix.  $\mathbf{0}_{3 \times 1}$  is the  $3 \times 1$  zero vector.  $\mathbf{F}_g$  is the gravity of the end-effector in  $F_e$ , and  $\mathbf{W}_g$  is the moment of the gravity about point  $P$  in  $F_e$ .  $\mathbf{F}_j$  is the  $3 \times 1$  joint force exerted on the joint by the end-effector. Moreover, one can write  $\mathbf{J}^T \boldsymbol{\tau}$  as

$$\mathbf{J}^T \boldsymbol{\tau} = \begin{bmatrix} \mathbf{F}_t \\ \mathbf{W}_t \end{bmatrix} \quad (17)$$

where  $\mathbf{F}_t$  is the resultant force of the cable tensions in  $F_e$ , and  $\mathbf{W}_t$  is the resultant moment of the cable tensions about point  $P$  in  $F_e$ .

Since the joint has three rotational DOFs, and the origin of the end-effector frame and the base frame are both set at the rotation center of the joint, one has  $\dot{\mathbf{X}} = [0 \ 0 \ 0 \ \dot{\theta}_x \ \dot{\theta}_y \ \dot{\theta}_z]^T$  and  $\ddot{\mathbf{X}} = [0 \ 0 \ 0 \ \ddot{\theta}_x \ \ddot{\theta}_y \ \ddot{\theta}_z]^T$ . Then the last three equations of (12) can be rewritten as

$$\mathbf{I}_p \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}_p \boldsymbol{\omega} + \mathbf{W}_g = -\mathbf{W}_t \quad (18)$$

where  $\boldsymbol{\omega}$  is the derivative of  $\boldsymbol{\omega}$ . The first three equations of (12) can be rewritten as

$$\mathbf{F}_j = -\mathbf{F}_t - \mathbf{F}_g - \boldsymbol{\omega} \times m\mathbf{v}_p \quad (19)$$

where (18) determines the rotation of the end-effector, and the joint force can be calculated based on (19).

## III. MOMENT EFFICIENCY INDEX OF A CDRD

According to (15), (17), and (19), the joint force generated by the CDRD is  $-\mathbf{F}_t$  and the joint force generated by the gravity of the end-effector is  $-\mathbf{F}_g$ . Moreover, the joint force generated by the centrifugal is  $-\boldsymbol{\omega} \times m\mathbf{v}_p$ . This paper defines a moment efficiency index  $\eta$  as the magnitude of the assistant moment of the CDRD (i.e.,  $|\mathbf{W}_t|$ ) divided by the magnitude of the joint force generated by the CDRD (i.e.,  $|\mathbf{F}_t|$ )

$$\eta = \frac{|\mathbf{W}_t|}{|\mathbf{F}_t|} \quad (20)$$

The characteristic length of  $\eta$  is set to 1 m in this study, so the unit of  $\eta$  is homogeneous [29]. The moment efficiency index reflects the ability of a CDRD to assist a joint while leading to a unit magnitude of joint force acting on the joint.  $\eta$  varies according to the orientation of the joint and the design of the CDRD.

Antagonistic cable tensions [30], [31] of a CDRD are cable tensions that balance each other. Since a joint with three rotational DOFs can balance any translational force in theory, antagonistic cable tension vector of the CDRD with three rotational DOFs, noted as  $\boldsymbol{\tau}_a$ , satisfies

$$\mathbf{J}^T \boldsymbol{\tau}_a = \begin{bmatrix} \mathbf{F}_a \\ \mathbf{0}_{3 \times 1} \end{bmatrix} \quad (21)$$

In this case, antagonistic cable tensions only balance the moment generated by each other. The resultant force generated by antagonistic cable tensions acts on the joint. It should be noted from (21) that, since antagonistic cable tensions create zero moment, they do not affect the rotation of the end-effector. However, antagonistic cable tensions do create a nonzero joint force, denoted as  $\mathbf{F}_a$ , acting on the joint. According to (21), in the force-closure workspace [21] of the CDRD, increasing antagonistic cable tensions does not affect the wrench acting on the end-effector, but it does increase the magnitude of the joint force generated by the CDRD. Thus, based on (20), the moment efficiency index will decrease.

## IV. A KNEE ASSISTED BY A CDRD WITH FOUR CABLES

In this section, a knee with one DOF (i.e., flexion and extension) assisted by an example three-DOF four-cable CDRD, as shown in Fig. 3, is discussed based on the moment efficiency index in quasi-static conditions. The collision between cables, cuffs, the shank, and the upper leg are not taken into account in this discussion. One cuff with a radius of 0.1 m is worn on the upper leg and the shank, respectively. The shank is able to rotate about  $x_e$  axis. The angle of the knee, denoted as  $\phi_k$ , is measured from the shank to  $z_b$  axis. Let  $d_u$  denote the distance between the centroid of the cuff worn on the upper leg and the knee joint. Let  $d_s$  denote the distance between the centroid of the cuff worn on the shank and the knee joint. Both  $d_u$  and  $d_s$  are variables in this study. Initially,  $F_e$  coincides with  $F_b$ , and the positions of the anchor points are shown in TABLE I.

According to (20) and (21), if the angle of the knee, the position of the upper leg cuff, and the position of the shank cuff are given, the maximum moment efficiency index, denoted by  $\eta_{max}$ , is achieved when the antagonistic cable tensions are all zero. In the simulation, the maximum moment efficiency index is calculated in the force-closure workspace of the CDRD. The maximum moment efficiency of the CDRD with respect to  $d_s$  and  $d_u$  in assisting the flexion and extension of the knee are plotted in Figs. 4 and 5, respectively.

According to Figs. 4 and 5, the angle of the knee and the distances from both cuffs to the knee joint significantly affect the maximum moment efficiency index. According to Fig. 4, when assisting the knee in flexion, the maximum moment efficiency index does not change much with the shank cuff moving closer to the knee (i.e.,  $d_s$  decreases) or the flexion of

the knee (i.e.,  $\phi_k$  increases). However, when the upper leg cuff is within 0.1 m from the knee ( $d_u \leq 0.1$  m), the maximum moment efficiency index decreases sharply with the upper leg cuff moving closer to the knee (i.e.,  $d_u$  decreases). As shown in Fig. 5, when assisting the knee in extension, the maximum moment efficiency index increases with the shank cuff moving closer to the knee (i.e.,  $d_s$  decreases), the upper leg cuff moving farther away from the knee (i.e.,  $d_u$  increases), or the flexion of the knee (i.e.,  $\phi_k$  increases).

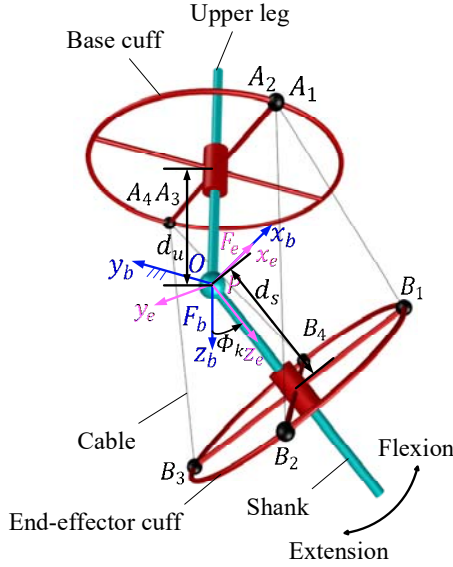


Fig. 3. Notations of a knee assisted by a three-DOF four-cable CDRD

TABLE I. POSITIONS OF ANCHOR POINTS (UNIT: M)

Anchor points	Positions of Anchor points on the base (upper leg)	Positions of Anchor points on the end-effector (shank)
1	$[0.1, 0.0, -d_u]^T$	$[0.0707, -0.0707, d_s]^T$
2	$[0.1, 0.0, -d_u]^T$	$[0.0707, 0.0707, d_s]^T$
3	$[-0.1, 0.0, -d_u]^T$	$[-0.0707, 0.0707, d_s]^T$
4	$[-0.1, 0.0, -d_u]^T$	$[-0.0707, -0.0707, d_s]^T$

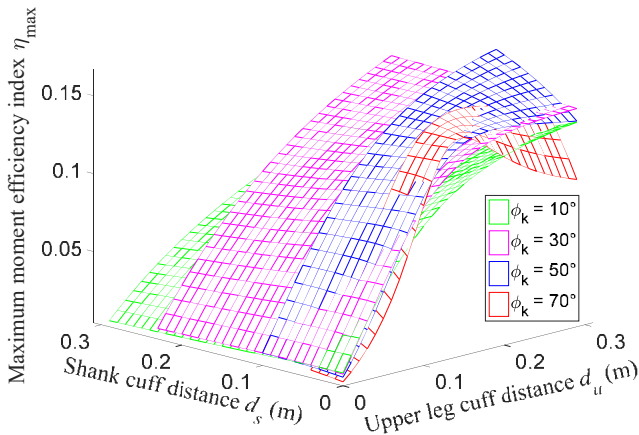


Fig. 4. The maximum moment efficiency index of the CDRD when assisting the flexion of the knee

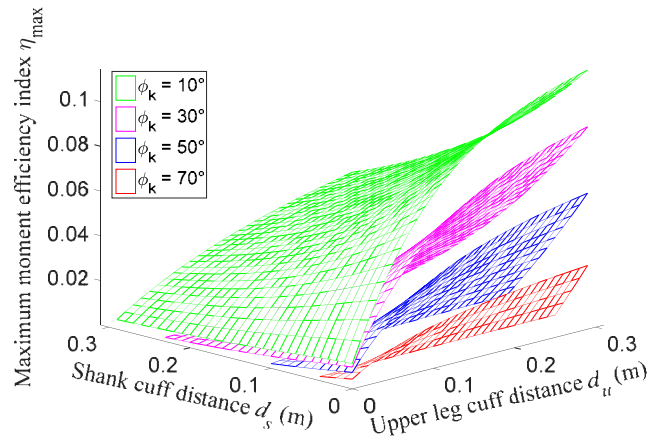


Fig. 5. The maximum moment efficiency index of the CDRD when assisting the extension of the knee

Based on Figs. 4 and 5, one can conclude that if the CDRD showed in Fig. 3 is used for knee rehabilitation or assistance, wearing the shank cuff close to the knee allows the CDRD to assist the knee with a large angle of the knee while wearing the upper leg cuff far away from the knee (up to about 0.15 m) can improve the moment efficiency of the CDRD. These findings on positioning the cuffs provide a cornerstone for further study (e.g., control and assistance strategies) of this CDRD.

## V. CONCLUSION

In this paper, the joint force generated by a CDRD in assisting a human joint was analyzed. The kinematics and dynamics models of a general three-DOF human joint assisted by a three-DOF four-cable CDRD were established first. Then, the moment efficiency index was proposed to quantitatively evaluate the efficiency of a CDRD in providing assistant moment to a human joint. A case study of a knee with one DOF assisted by a three-DOF CDRD with four cables was conducted based on the moment efficiency index. The moment efficiency index proposed in this paper not only promotes safety of rehabilitation training, but also provides a guideline to for the design of CDRDs.

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