

Enhancing Biped Locomotion on Unknown Terrain Using Tactile Feedback

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Abstract—Human bipedal balance during standing and walking depends on several receptors including the cutaneous receptors in the glabrous skin of the foot sole. It has been shown in human-involved studies that the different areas of the sole have distinct sensitivities and serve a different purpose in both walking and standing. In humanoid robotics, the feedback to keep balance is mainly achieved using force-torque sensors mounted at the robot's ankles. Although these sensors can accurately estimate the center of pressure of a foothold, they cannot provide information about the pressure shape of the footprint and therefore can miss ill terrain conditions during locomotion. In this paper, we present a biologically inspired sole skin sensor based on the robot skin developed at our lab. The robot skin can enhance and complement the ankle force-torque sensors used in balancing and walking controllers by providing additional information that a force-torque sensor cannot produce. This additional information can be used to reconstruct the supporting polygon and the pressure footprint online. We present a case study where a force-torque sensor fails to detect the terrain conditions while the skin succeeds and the information is used to re-plan the footsteps position.

I. INTRODUCTION

Humanoid robots are developed with the aim of executing tasks that are originally performed by human beings. Tasks such as personal service, industrial manufacturing or disaster relief require a robot to be capable of navigating on complex environments while carrying out other tasks. The terrain in these scenarios may be compounded by a random set of movable and fixed obstacles, ramps, stairs or any kind of unknown obstacles that the robot must step on or avoid according to its capabilities.

A. Humanoid Locomotion on Uneven Terrain.

For a humanoid robot, locomotion itself is a complex task due to its human-shaped architecture with unstable dynamics. A stable bipedal gait is needed when the upper limbs are required for other purposes. When the terrain is structured, known, and therefore the footholds can be predefined, walking can be achieved using a Model Predictive Controller as in [1], or with optimal trajectories as in [2].

When the terrain is unstructured and unknown, the locomotion paradigm must be adapted to acquire information online and react to unexpected conditions. This has been done, for example, using a perception-based approach in [3],

where the terrain is scanned in order to search for suitable footholds. The footsteps are planned ahead considering both the position and orientation of the footholds. In [4] the robot uses exploratory foot motions looking for edges on the terrain and finding the best foot pose to hold the next steps. In [5] the foot orientation is adapted using feedback from a force-torque sensor (FT sensor) in the ankle of the robot. [6] and [7] present algorithms that use IMU sensors and FT sensors to deal with unknown terrain inclination. [8], [9] and [10] use contact switches mounted on the robot soles to detect premature ground contact and adapt online with motion primitives to the new terrain conditions.

B. A Biologically Inspired Approach.

The postural equilibrium of a human being depends on a complex fusion of vestibular, visual, proprioceptive and exteroceptive receptors. While it is known that none is essential for balance, the lack of each one produces different affectation into the posture and motions. Past works on humanoid robotics have approached the bipedal balance problem by using sensors to gather similar information like the human body's receptors, for example, visual information can be acquired by camera systems, the vestibular receptors can be emulated with IMU sensors, proprioceptive with encoders, and FT sensors have been collocated in the feet as exteroceptive receptors. The use of the FT sensors on the robot feet enables the estimation of the center of pressure and contact forces in an indirect manner that is accurate enough for many applications. However, the human foot has a glabrous skin that enables the acquisition of more information about the terrain such as the texture, the stability, the temperature and the pressure distribution on the foothold. With all this information, the human foot can detect complex terrain conditions as inclination, vibration, slipperiness among others.

The role of sole cutaneous receptors in the human equilibrium has been studied for more than one century [11], it has been proved that subjects with anesthetized soles present considerable difficulties to keep their standing equilibrium. In [12] a similar experiment was performed on subjects with the soles anesthetized by hypothermia who were blindfolded to induce posture sway. The induced posture sway was amplified considerably when the sensitivity of the barefoot sole is artificially reduced. The same principle was probed in [13] inducing posture sway galvanically on the subjects.

Apart from its paramount role in standing equilibrium, the cutaneous receptors of the foot sole have also an important function in the walking process. As probed in [14] and [15],

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they are used to regulate the timing of the events in the gait process and the phase of the motion cycles. In their experiments, the walking patterns of the subjects changed when artificial stimuli were induced in the cutaneous receptors in the sole. These receptors are distributed with different densities within the sole as shown in [16]. The different areas of the sole have different sensitivities to pressure, they are also connected to different muscles that generate different changes in the gait pattern when stimulated.

The absence of tactile receptors, as in the case of an amputee, forces the patient to replace the tactile feedback signals to adjust his/her gait by other means, for example compensating with the proprioceptive nerves or relying more on visual and vestibular feedback. This reduces the patient's ability to avoid small obstacles as shown in [17]. Apart from the delays produced in the gait cycle by the absence of tactile feedback, it also affects the task of mapping the terrain to adapt the foothold online and makes the patient prone to stumble. These effects are also visible due to cutaneous sensitivity loss due to aging or the development of pathological diseases [18].

C. More Senses for Humanoid Robots.

Bipedal balance depends highly on force feedback during both standing and walking. This has been implemented using FT sensors mounted in the feet of humanoid robots. Most balance controllers use FT sensors to estimate the center of pressure (CoP) and the zero moment point (ZMP) [19] for the balance feedback loop. The FT sensors also have been used to adjust the timing for the walking cycles [9], [20], [10] and to evaluate the conditions of the terrain using techniques as exploratory motions [4]. However, the estimations of the CoP using FT sensors in stationary stances uses the assumption that the complete sole is in contact with the ground but there is no precise information on the shape of the pressure distribution on the sole. In this paper, we introduce a new sensing approach to enhance the walking process based on the robot skin [21]. Providing tactile sensing capabilities to the foot sole of a humanoid robot, it is possible to get not only the online estimation of the CoP and ZMP but also the shape and area of the contact surface at the instant of the foot landing. We will present a case study of a small obstacle avoidance during quasi-static walking where the lack of information leads a standard FT sensor to assume healthy stepping conditions where a premature contact is detected.

The remaining sections are arranged as follows: in Section II, the sole tactile sensor is detailed and its features are specified, Section III will present the case study where we compare the robot skin and the FT sensor, in section IV final comments and conclusions will be presented.

II. SOLE TACTILE SENSING.

In recent years, robotic tactile sensing gained attention for both industrial [22] and research-oriented platforms like humanoid robots [23]. Robot skin can provide precise information about external contacts with the environment in other

parts of the body than the end effectors. This information can be used to improve human-robot interaction and to enable practical ways of active compliance and environment exploration.

The weight of a full-size humanoid robot can be around 100 Kg, therefore, when the robot is in a single support phase, the skin sensor mounted on the sole must be capable of holding such weight and still sense variations in the pressure distribution. The force range in many robot skin sensors was designed for soft interactions and is not suitable for high loads. However, the robot skin developed at our lab can support an 80 Kg person while jumping on a single cell [21], [24]. Based on this results, we have designed a 42 Skin-Cell patch to cover each sole of a humanoid robot, see Figure 1. The skin on the sole can measure normal contact forces, temperature, 3-axis acceleration and proximity (pre-contact up to 100 mm), as shown in Figure 2. The complete tactile information can be delivered at a rate up to 250 Hz in large scale areas (covering not only the soles) due to new neuromorphic paradigms, such as the event-driven communication architecture [26].

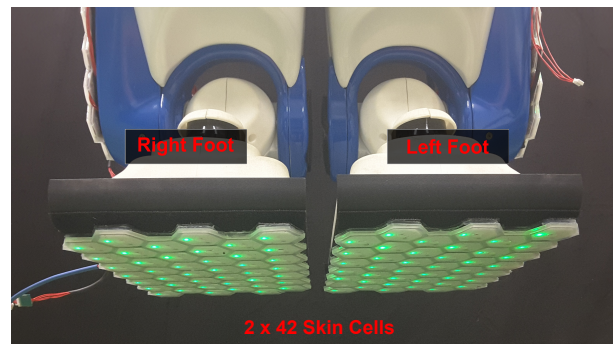


Fig. 1: Robot skin mounted on the soles of the REEM-C humanoid robot [25].

The robot skin architecture provides the position of every cell on the sole [27]. This information can be used to reconstruct the pressure shape when the foot is on the ground. Following the biological principle of the sole skin in human feet, a thicker external layer was molded for these patches to enlarge the durability. The silicon material of the skin patch shows high friction coefficient over different surfaces. This improves the stability of the steps, reducing the likelihood of slipping. The skin network automatically reconfigures its topology in case some cells get damaged during a harmful interaction. This makes it feasible to apply robot skin on the soles, with robustness to the rough conditions that walking on uneven terrain can generate.

A. Center of Pressure Measurement.

There are different manners to estimate the center of pressure of a robot sole [28]. One is using multiple single-axis force sensors distributed on the sole. Our robot skin accomplishes this requirement. It allows covering the whole area of the sole with a uniform distribution. Then, for a single

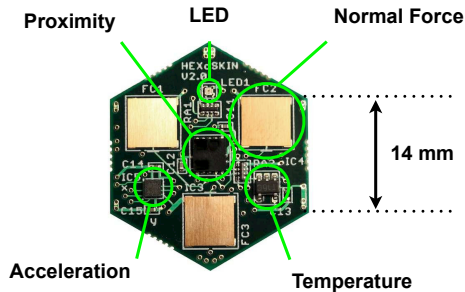


Fig. 2: Sensing modalities of the robot skin [21], [24]. Every skin cell has a 3-Axis accelerometer, a proximity sensor, 3 normal force sensors, and a temperature sensor.

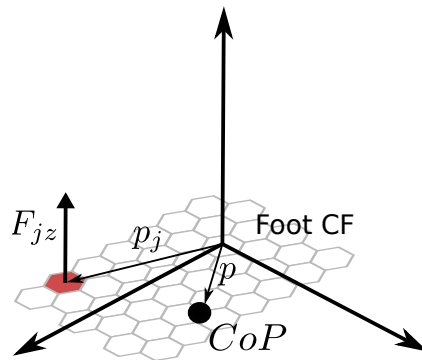


Fig. 3: CoP estimation using the skin information. Foot CF is the reference frame of the foot.

sole, the center of pressure is defined as

$$p_x = \frac{\sum_{j=1}^N p_{jx} F_{jz}}{\sum_{j=1}^N F_{jz}} \quad (1)$$

$$p_y = \frac{\sum_{j=1}^N p_{jy} F_{jz}}{\sum_{j=1}^N F_{jz}}, \quad (2)$$

where p_x and p_y are the CoP's position coordinates, p_{jx} and p_{jy} are the position coordinates of the j -th skin cell in a patch with N cells, and F_{jz} the measured vertical force in the j -th skin cell as shown in Figure 3. In Figure 4, a comparison between the estimation of the CoP using an FT sensor in the ankle [28] and using the skin information (Eqn. 1-2) is shown. Although some skin technologies can only measure normal forces, for the non *foot edge contacts* case “flat foot”, they can estimate the CoP without the need of estimation techniques, e.g. foot tilt estimation. In this case, the skin sensor can replace the FT sensor. In the case of *edge contacts* (foot tilting), the dynamics of the robot can be used to compensate the unmeasured components of the contact forces. The inertial sensors in our skin can help for this purpose. Another solution is to combine the skin information with a direct measurement from an FT sensor, where the skin sensor provides the number and location of the contact points and the FT sensor provides the total wrench at the ankle. In Figure 4, we can observe that the estimated CoP from the skin sensor matches the FT sensor estimation without any compensation. Only in the cases when the foot tilts, the estimation deviates with an error around 10% of the CoP displacement (see zoomed section). This can also be observed in the companion video. The robustness of the skin system allows keeping an accurate estimation of the Cop even when some of the cells are damaged, just losing the components of the dead sensors but keeping the overall estimation in real time.

B. Supporting Polygon Reconstruction

The spatial distribution of the force sensors can be exploited to map the shape and area of the contact in the foothold. With this information, the supporting polygon of the foot can be reconstructed online. A threshold can be set for the measured contact force to consider that a skin

cell is contributing to holding the pressure footprint. After traversing all the cells in the patch, the convex hull containing all the contact points can be reconstructed to define the supporting polygon of the foot using classic algorithms as in [29]. The dataset can also be used to build a concave hull as in [30] to reconstruct a precise shape of the foothold even before hitting the ground thanks to the pre-touch modality of the robot skin as shown in Figure 5. The pre-touch reconstruction can be used as preemptive information, which represents a prediction of the supporting polygon. However, the correct pressure footprint can only be acquired during the contact. As the prediction is taken directly from the surface of the sole, it is not prone to occlusion neither for forwarding or backward gaits as it would be when mounting lidar sensors or depth cameras on the knee.

III. CASE STUDY

To demonstrate the advantages of using robot skin rather than only FT sensors in the ankles of the foot during locomotion, an exploration walking scenario is proposed. For this test, the robot is intended to walk on a straight line for a few steps, the robot will not have previous information about the terrain. The walking controller is similar to the one used in [8]. For every step, the swing foot trajectory is planned using a spline that is supposed to finish at zero height (ground level).

During the swing foot trajectory, a premature contact condition is defined as $\|F_s\| \geq \epsilon_f$ (see Figure 6). This means that the foot collided with an obstacle and therefore the motion primitive is stopped immediately. At the moment of the collision, the ankle-torque norm is compared to a threshold $\|M_s\| \geq \epsilon_t$ to detect if the obstacle is safe to step on. If the detected obstacle is considered safe to step on, the robot continues to walk over it. For non-safe obstacles, the robot replans the footstep and steps a few centimeters away to avoid the obstacle. The threshold ϵ_t was defined as the torque generated when the foot touches down on flat terrain.

In the experiment, the FT sensor successfully identifies the obstacles that generate enough torque in the ankle $\|M_s\| \geq \epsilon_t$ (case (d) from Figure 7). However, the torque generated in cases (a), (b) and (c) is similar to the one generated by flat ground. This is safe for cases (a) and (b) but leads

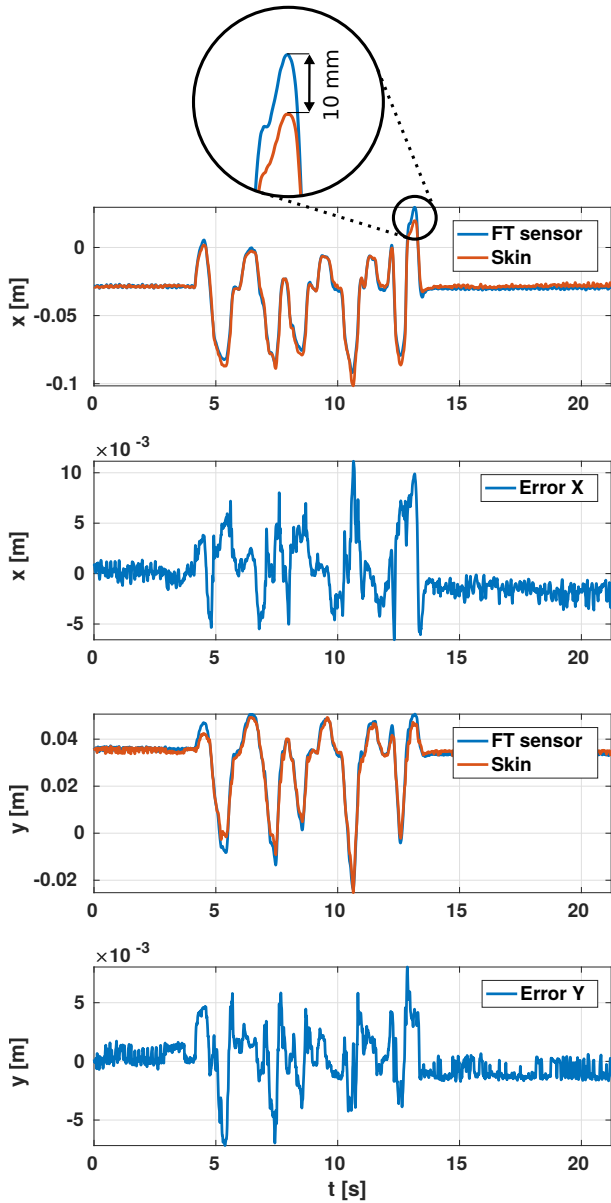


Fig. 4: Left foot CoP estimation using an FT sensor in the ankle and the sole skin. The robot was standing and was pushed to produce changes in the center of pressure. When the foot tilts, the difference is bigger because it was not compensated in the estimation using skin.

to falling in case (d). This case study probes that with a single instantaneous contact, FT sensors cannot detect unsafe terrain conditions. With incomplete information, a walking controller can make incorrect assumptions and fail to keep balance.

The robot skin, on the other hand, can provide additional information with a single instantaneous contact to detect the unsafe terrain condition. In this example, another safety check was implemented before resuming the walking. With the reconstruction of the supporting polygon, the robot can know the percentage of the area of the sole that is in contact with the ground. The ratio between the measured supporting

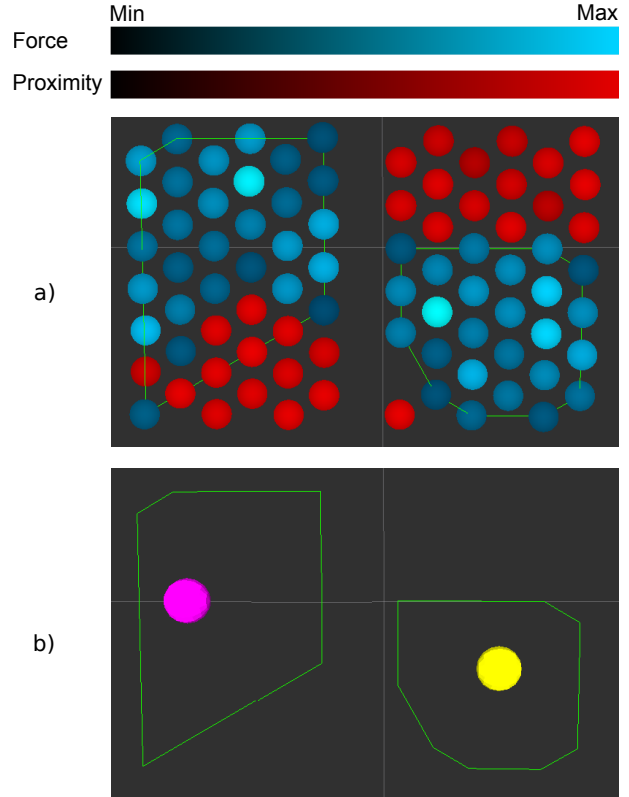


Fig. 5: Reconstruction of the supporting polygon and the CoP from tactile information. The robot was standing over non-flat terrain. a) Pressure footprint from ROS-RVIZ visualization. Red and blue markers represent proximity and contact forces respectively. b) Reconstruction of the convex hull (supporting polygon) and CoP of each foot. The robot skin can detect accurately the floor irregularities.

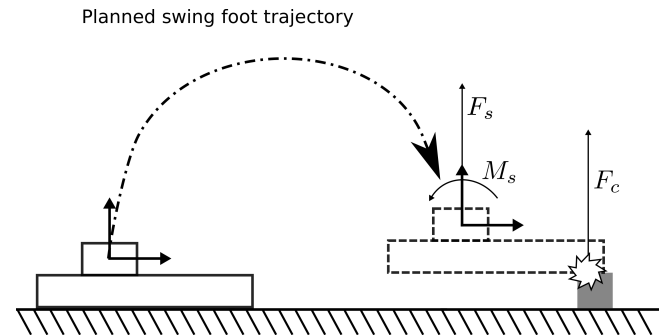


Fig. 6: The motion of a foot during one step. When a premature contact is detected $\|F_s\| > \epsilon_f$, the step re-plan is evaluated to define if it is safe to step on the detected obstacle.

polygon area A_p and the total sole area A_s is compared to a safe area threshold as $\frac{A_p}{A_s} > \epsilon_A$. With this improvement, the robot can detect successfully any small obstacle to replan the footstep, and detect the big obstacles as safe to step on to continue walking.

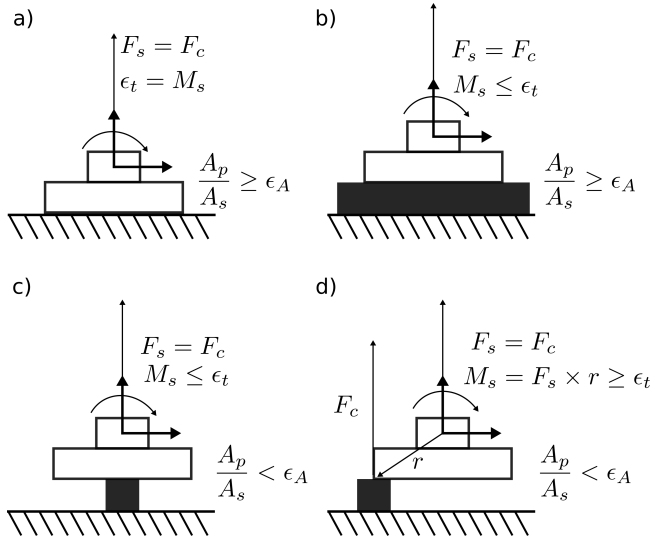


Fig. 7: When foot contact is detected, the forces and torques are evaluated. a) Foot landing on flat terrain, this condition is used to define ϵ_t . b) Foot landing on a large flat obstacle. c) Foot landing on a thin flat obstacle aligned to the foot and centered under the ankle. d) Foot landing on a thin flat obstacle, not aligned to the ankle.

The experiment was reproduced with four different obstacles: i) a wooden board, ii) a non-centered wooden bar, iii) a centered wooden bar (aligned to the ankle FT sensor’s axes), and iv) a complex shape obstacle made with soft plastic balls. These results are reported in Table I. The FT sensor accomplished to detect the wooden board to step on, and the non-centered bar to trigger a step re-plan. However, the FT sensor detected the centered bar and the complex obstacle as safe to step on, leading the robot to fall. The skin sensor succeeded to detect all the unsafe small obstacles and triggered the re-plan as shown in Figure 8. However, it also detected the wooden board as safe to step on.

IV. CONCLUSION.

Balance during biped locomotion relies on precise force feedback due to its unstable dynamics. In human beings, this feedback is provided by cutaneous and proprioceptive receptors. These cutaneous receptors are distributed all over the soles and provide different sensitivities for each area. These receptors provide information not only about the main supporting force and ground reaction force but also a detailed shape of the pressure distribution of the footprint. With this information, we modify our foot’s position and orientation during both standing and locomotion on uneven terrain.

Past works have studied the force feedback for biped balance mainly through FT sensors mounted at the ankle.

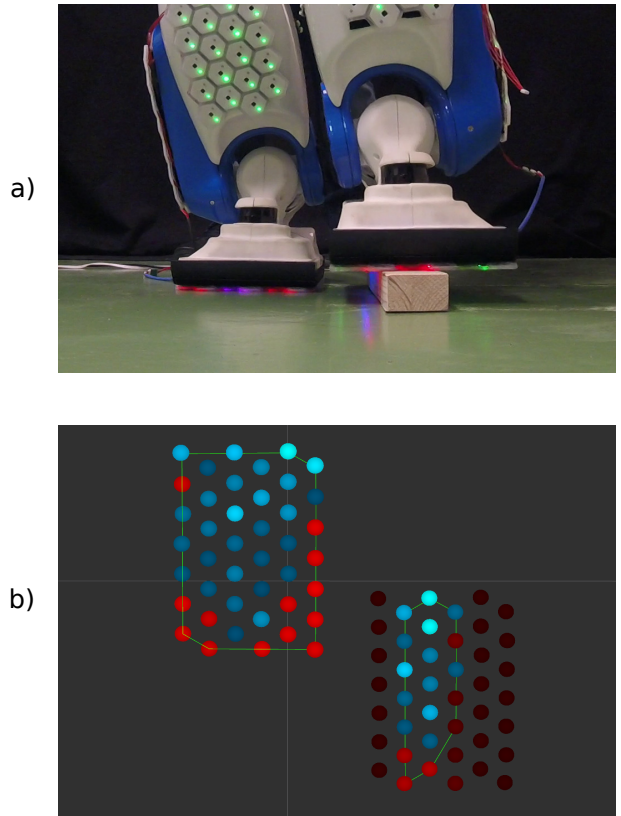

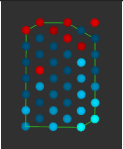

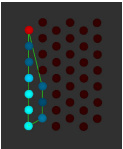

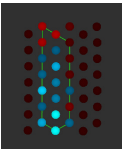
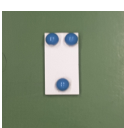
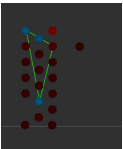


Fig. 8: Experimental results when the robot detects a thin flat obstacle that generates ankle torques similar to flat ground. a) The moment when a premature contact occurs produced by the collision of the foot with the obstacle. b) The reconstructed polygon at the moment of the premature contact.

This setup can be used to measure the position of the CoP and ground reaction forces needed for balance controllers. However, the use of FT sensors cannot assess the shape of the contact area instantaneously. This information can be obtained by other means, e.g., including external sensors (vision systems or contact switches), or using exploratory motions that depending on the terrain can take longer times.

In this paper, we proposed the use of a skin tactile sensor that can estimate the CoP needed for balance controllers similar to the classic ankle FT sensors, probing that these FT sensors can be enhanced or complemented by the robot skin. Additionally, the tactile (touch and pre-touch) information from the robot skin can provide instantaneous information about the shape of the footprint that can be used to assess the terrain conditions at foot landing without the need of exploratory motions or explosive motions from the ankle, which require fast dynamics with powerful actuators. This instantaneous information allows generating fast reactive and preemptive motions. In the presented case study, the tactile information was used to replan the step looking for a bigger supporting polygon area with fixed foot orientation. Future work will explore more dynamic reactions to the tactile stimuli for dynamic walking on uneven terrain. This new

TABLE I: Tested Obstacles

Obstacle	Footprint	$\ M_t\ $ $\epsilon_t = 8.5$	Sorted by FT sensor	Sorted by Skin
		8.0	Stepping on obstacle	Stepping on obstacle
		8.6	Footstep re-plan	Footstep re-plan
		6.0	Stepping on obstacle	Footstep re-plan
		4.5	Stepping on obstacle	Footstep re-plan

Green: Robot completed the test.

Red: Robot fell down.

approach of terrain sensing opens the door to a wider span of capabilities for locomotion on uneven terrain, even for simple robots and controllers without high dynamic capabilities.

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