Autonomous Biped Stepping Control Based on the LIPM Potential

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Abstract— This paper proposes a novel biped stepping control which does not depend on the time-defined trajectory. The updown motion of the foot is determined by referring to the LIPM potential, which is defined in the paper, based on the phase-space analysis of the center of mass (COM). The motion rate of the COM relative to the Zero-Moment Point (ZMP) is represented as the gradient of the potential, and the potential monotonously decreases from positive to negative during one step. The dominant component of the COM movement turns from convergent mode to divergent mode at the zero potential, and thus, the positive potential encourages the lifting-up and the negative potential alerts the necessity of touch-down. This emerges a stable alternate stepping of the feet by combining with a self-exciting oscillation of the COM and the ZMP, which was also proposed by one of the authors. The controller provides robots with flexibility against disturbances since it does not rely on any pre-defined referential motion trajectory. Computer simulations show that this idea is valid for a bipedal foot controller.

I. INTRODUCTION

Biped robots that locomote in real environments should have high flexibility against various unpredictable events. It is hardly achievable by a control scheme that relies on time-driven referential trajectories, which is adopted in many robots [1], [2], [3]. A challenge is how to synthesize the whole-body motion ruled by a complex dynamics without off-line computations.

The model predictive control is one of the promising approaches since it can produce the optimal motion in realtime with discontinuous change of the contact state taken into account, and has been applied in many works [4], [5], [6]. A drawback is that it is less robust against the mismatch between the planned and the actual contact states. It is also unpreferable that it requires a heavy computation. Other methods presented so far include hybrid zero-dynamics [7], passive dynamic walking [8], and nonlinear oscillators [9], while they have difficulties in embedding intended locomotion patterns other than straight walking into them.

Sugihara [10] proposed another control scheme for biped robots. It is based on a simplified dynamics between the center of mass (COM) and the Zero-Moment Point (ZMP) [11], where the ZMP is handled as an indirect input [12]. A nonlinear feedback of the COM state to the desired ZMP location emerges a stable limit cycle, in which the stepping

²Tomomichi Sugihara is with Department of Adaptive Machine Systems, Graduate School of Engineering, Osaka University, 2-1 Yamadaoka, Suita, 565-0871 Japan zhidao@ieee.org period and stride are easily adjustable. The maximum stability is secured by directly taking into account the unilateral constraint on the reaction force, i.e., the constraint on the ZMP within the supporting region. It has been enhanced to the omnidirectional walking [13], emergent step-out [14] and even jumping [15].

The idea of stepping control in the above is as follows. When the COM converges to the stable limit cycle, the locus of the ZMP is predictable based on the frequency response. Hence, a foot stepping which is consistent with the stationary oscillation can be achieved through the observation of the ZMP - the robot starts to lift up the foot after the ZMP comes into the pivot sole, and touches it down when the ZMP is about to go out of the pivot sole. A problem found in a later study is that the robot under the above control reacts sensitively to perturbations, as explained in Section II. The authors have concluded that it is inappropriate to define the foot-lifting motion based on the location of the ZMP moving in a narrow pivot sole.

This paper presents a novel biped stepping control. A phase-space analysis of the COM suggests that the LIPM potential, which is also defined in this paper, can be a criterion to determine the referential velocity of the foot. The motion rate of the COM relative to the ZMP is represented as the gradient of the potential, and the potential monotonously decreases from positive to negative during one step. The dominant component of the COM movement turns from convergent mode to divergent mode at the zero potential, and thus, the positive potential encourages the lifting-up and the negative potential alerts the necessity of touch-down. This idea was implemented in computer simulations, which showed that the robot stability against external perturbations was upgraded.

II. BIPED STEPPING REFERRING TO ZMP OSCILLATION AND ITS DRAWBACK

A. COM-ZMP Oscillator By Non-linear Feedback

This section explains the previous controller [10] to be improved in this paper. The discussion goes based on the COM-ZMP model [12]. Let us assign x-, y- and z-axes along with the longitudinal, lateral and vertical directions, respectively, as depicted in Fig. 1, and denote the locations of the COM and the ZMP are defined by $\boldsymbol{x} = \begin{bmatrix} x & y & z \end{bmatrix}^{T}$ and $\boldsymbol{x}_{Z} = \begin{bmatrix} x_{Z} & y_{Z} & z_{Z} \end{bmatrix}^{T}$, respectively. Suppose the inertial torque about the COM of the robot is negligible and the height of the COM $z_{0} = z - z_{Z}$ is almost constant for simplicity, the following simplified equation of motion is

^{*}This work was supported by Grant-in-Aid for Scientific Research(B) (#18H03310) from Japan Society for the Promotion of Science.

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Fig. 1. COM-ZMP model and definition of axes

obtained:

$$\ddot{x} = \zeta_0^2 (x - x_{\rm Z}) \tag{1}$$

$$\ddot{y} = \zeta_0^2 (y - y_{\rm Z}) \tag{2}$$

$$\zeta_0 \stackrel{\text{def}}{=} \sqrt{\frac{g}{z_0}} : \text{const.},\tag{3}$$

where $g = 9.8 \text{m/s}^2$ is the acceleration due to the gravity. An important constraint that the ZMP x_Z lies within the supporting region S is posed as

$$\boldsymbol{x}_{\mathrm{Z}} \in \mathcal{S}.$$
 (4)

The biped motion requires a combination of manipulation of the ZMP within S and discontinuous deformation of S. Note that Eqs. (1) and (2) are symmetric with respect to x and y, so that only the motion in y-axis is considered hereafter in this section. In order to emerge a stable biped stepping motion, the ZMP should move alternately from one sole to another, and the vertical foot motion should synchronize with it, whereas the stability of the COM should be guaranteed. The following feedback controller enables this requirement:

$$y_{\rm Z} = \begin{cases} y_{\rm Zmax} & ({\rm S1} : \bar{y}_{\rm Zmax} < y_{\rm Z}) \\ \bar{y}_{\rm Z} & ({\rm S2} : y_{\rm Zmin} \le \bar{y}_{\rm Z} \le y_{\rm Zmax}) \\ y_{\rm Zmin} & ({\rm S3} : \bar{y}_{\rm Z} < y_{\rm Zmin}) \end{cases}$$
(5)

$$\bar{y}_{\mathrm{Z}} = {}^{\mathrm{d}}y + (q_y + 1)\left(y - {}^{\mathrm{d}}y + f(\gamma)\frac{\dot{y}}{\zeta_0}\right) \tag{6}$$

$$f(\gamma) \stackrel{\text{def}}{=} 1 - \rho \exp k \left(1 - \frac{(q_y + 1)^2 \gamma^2}{r^2} \right) \tag{7}$$

$$\gamma \stackrel{\text{def}}{=} \sqrt{(y - {}^{\mathrm{d}}y)^2 + \frac{\dot{y}^2}{\zeta_0^2 q_y}},\tag{8}$$

where ${}^{d}y$ is the referential position of the COM, and $q_y > 0$, r > 0 and k > 0 are control parameters. q_y defines the frequency of the oscillation. r is the nominal half distance between the feet. Suppose the actual ZMP is manipulated to follow the above desired ZMP without an error, the dynamics of the COM is represented by a piecewise autonomous system as

$$\ddot{y} = \begin{cases} \zeta_0^2 y - \zeta_0^2 y_{\text{Zmax}} & (S1) \\ -\zeta_0 (q_y + 1) f(\gamma) \dot{y} - \zeta_0^2 q_y (y - {}^{\mathrm{d}}y) & (S2) \\ \zeta_0^2 y - \zeta_0^2 y_{\text{Zmin}} & (S3) \end{cases}$$



Fig. 2. Phase portrait of the system represented by Eq. (9)

When $\rho = 0$, the controller is identical to the stabilitymaximized COM-ZMP regulator [16]. The system with $\rho > e^{-1}$ has the following stable limit cycle in (S2), which is

$$(y - {}^{\mathrm{d}}y)^2 + \frac{\dot{y}^2}{\zeta_0^2 q_y} = \left\{\frac{1}{(q_y + 1)^2} + \log\rho\right\} r^2.$$
(10)

For $\rho = 1$, this is a harmonic oscillation with the amplitude $r/(q_y+1)$ and the period $2\pi/\zeta_0\sqrt{q_y}$. Fig. 2 shows the phase portrait of the system Eq. (9).

B. Phase-driven Stepping Control and its drawback

The frequency response from y_Z to y tells that the ZMP is synchronized with the COM oscillating on the harmonic limit cycle, where the amplitude is r. This fact means that the ZMP in the stationary state periodically reciprocates between the both feet and its locus is predictable. Then, the up-down motion of the foot can be determined by abstracting the phase information from the ZMP oscillation as follows.

In order to avoid the problem of ZMP jumping, the phase is defined by the following complex number:

$$p_{\rm Z} \stackrel{\rm def}{=} \left(y_{\rm Z} - {}^{\rm d}y \right) - \frac{(q_y + 1)\dot{y}}{\zeta_0 \sqrt{q_y}} i,\tag{11}$$

where *i* is the imaginary unit. The real part is the position of the ZMP. The imaginary part implies the time-derivative of the ZMP movement, while the COM velocity is used instead of the deviation of the ZMP since the trajectory of the ZMP is not necessarily differentiable by time. Let us consider the locus of p_Z in Fig. 3. It is segmented at two points p_{Lin} and p_{Lout} along a trajectory, the real parts of which are both the inner edge of the left sole y_{Lin} during one step. The right foot is liftable when p_Z is in this segment. An idea to control the right foot is to detach off the ground when $p_Z = p_{\text{Lin}}$ and touch down when $p_Z = p_{\text{Lout}}$. Although those two points cannot be detected in advance, their estimates, \bar{p}_{Lin} and \bar{p}_{Lout} , can be obtained if $|p_Z| > y_{\text{Lin}}$ as

$$\bar{p}_{\rm Lin} = y_{\rm Lin} - i\sqrt{y_{\rm Z}^2 - y_{\rm Lin}^2}$$
 (12)

$$\bar{p}_{\rm Lout} = y_{\rm Lin} + i \sqrt{y_{\rm Z}^2 - y_{\rm Lin}^2}.$$
 (13)

As the COM converges to the limit cycle, they also asymptotically converge to the actual p_{Lin} and p_{Lout} , respectively,



Fig. 3. Estimation of the start and end values of p_Z in foot-liftable phase

as depicted in Fig. 3. It is also noticed that the ZMP lies within the left sole as long as $p_{\rm Z}$ satisfies

$$0 < \theta_{\rm L} < 1 \tag{14}$$

$$\theta_{\rm L} \stackrel{\rm def}{=} \frac{\angle (|p_{\rm Z}|/\bar{p}_{\rm Lin})}{\angle (\bar{p}_{\rm Lout}/\bar{p}_{\rm Lin})},\tag{15}$$

where $\theta_{\rm L}$ provides the phase information for the foot-lifting since it increases from 0 to 1 during the step. If and only if $|p_{\rm Z}| > y_{\rm Lin}$ is satisfied, the referential height of the right foot ${}^{\rm d}z_{\rm FR}$ is defined, for example, as

$${}^{\mathrm{d}}z_{\mathrm{FR}} = \frac{1}{2} \frac{h|p_{\mathrm{Z}}|}{r} \sigma(\rho) (1 - \cos 2\pi\theta_{\mathrm{L}}) \tag{16}$$

$$\sigma(\rho) \stackrel{\text{def}}{=} \begin{cases} 1 & (\rho > 1) \\ \frac{\rho - e^{-1}}{1 - e^{-1}} & (e^{-1} \le \rho \le 1) \\ 0 & (0 \le \rho \le e^{-1}) \end{cases} , \quad (17)$$

where *h* is the nominal lifting height. Otherwise, ${}^{d}z_{\rm FR} = 0$. The maximum lifting height depends on $|p_Z|/r$, which measures the degree of convergence to the limit cycle. Concerning with the horizontal movement of the lifting foot, the referential position ${}^{d}x_{\rm FR}$ and ${}^{d}y_{\rm FR}$ are provided based on the capturability [17] as

$$\begin{bmatrix} ^{\mathrm{d}} x_{\mathrm{FR}} & ^{\mathrm{d}} y_{\mathrm{FR}} \end{bmatrix}^{\mathrm{T}} = \mathbf{sat}(\begin{bmatrix} x_{\mathrm{ICP}} & y_{\mathrm{ICP}} \end{bmatrix}^{\mathrm{T}}, \mathcal{F}_{\mathrm{R}}), \quad (18)$$

where $\operatorname{sat}(\begin{bmatrix} x_{\operatorname{ICP}} & y_{\operatorname{ICP}} \end{bmatrix}^{\mathrm{T}}, \mathcal{F}_{\mathrm{R}})$ is a function that returns the proximity of $\begin{bmatrix} x_{\operatorname{ICP}} & y_{\operatorname{ICP}} \end{bmatrix}^{\mathrm{T}}$ to a 2-dimensional region $\mathcal{F}_{\mathrm{R}}, \mathcal{F}_{\mathrm{R}}$ is the nominal reachable range of the right foot, and

$$x_{\rm ICP} \stackrel{\rm def}{=} x + \frac{\dot{x}}{\zeta_0} \tag{19}$$

$$y_{\rm ICP} \stackrel{\rm def}{=} y + \frac{\dot{y}}{\zeta_0}.$$
 (20)

The above $\begin{bmatrix} d x_{FR} & d y_{FR} \end{bmatrix}^T$ is determined such that the COM regains the stability as soon as the foot lands on the ground, and the saturation within the reachable range prevents the robot from excess step-out and self-collision.

In the actual robot controller, the desired position of the ZMP defined by Eq. (5) is converted to the equivalent COM position with a disturbance observer, and the referential



Fig. 4. The LIPM potential in phase space

movement of the lifting foot is determined from Eqs. (16) and (18). The robot foot follows the referential position by a PD controller as

$$\ddot{\boldsymbol{x}}_{\mathrm{FR}} = -\boldsymbol{K}_{\mathrm{p}}(\boldsymbol{x}_{\mathrm{FR}} - {}^{\mathrm{d}}\boldsymbol{x}_{\mathrm{FR}}) - \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{x}}_{\mathrm{FR}},$$
 (21)

where $\mathbf{x}_{\text{FR}} = \begin{bmatrix} x_{\text{FR}} & y_{\text{FR}} & z_{\text{FR}} \end{bmatrix}^{\text{T}}$, $d\mathbf{x}_{\text{FR}} = \begin{bmatrix} dx_{\text{FR}} & dy_{\text{FR}} & dz_{\text{FR}} \end{bmatrix}^{\text{T}}$, \mathbf{K}_{p} and \mathbf{K}_{d} are the position of the right foot, the referential position of the stepping foot and the gain matrices, respectively. The acceleration of the left foot $\ddot{\mathbf{x}}_{\text{FL}}$ is also defined in a symmetric way. They are converted to the whole joint displacements through the inverse kinematics, and the joints are controlled to track them.

The above idea works under certain magnitude of disturbances. However, when the robot is strongly pushed, the ZMP reaches the inner edge of the pivot sole earlier than the regular situation. In such situations, the robot prefers to land on the ground rather than to largely step out due to the above phase-driven control, which decreases the stability. This behavior is caused since the vertical foot motion depends on the relative position of the ZMP in a narrow pivot sole and even a small deviation of the ZMP largely influences the lifting height. Hence, another reference to determine the foot motion than the above ZMP-based phase should be found. The author's idea for the new reference is the LIPM potential, which is explained in the next section.

III. THE LIPM POTENTIAL AND BIPED STEPPING CONTROL

A. The LIPM Potential

Let us define the dimensionless position of the COM, time as

$$\tilde{\boldsymbol{r}} = \begin{bmatrix} \tilde{x} & \tilde{y} & \tilde{z} \end{bmatrix}^{\mathrm{T}} \stackrel{\mathrm{def}}{=} \begin{bmatrix} x - \bar{x}_{\mathrm{Z}} & y - \bar{y}_{\mathrm{Z}} \\ z - \bar{z}_{\mathrm{Z}} & z - \bar{z}_{\mathrm{Z}} \end{bmatrix}^{\mathrm{T}}$$
(22)
$$\tilde{t} = \zeta_0 t,$$
(23)

where $\bar{\boldsymbol{x}}_{Z} = \begin{bmatrix} \bar{x}_{Z} & \bar{y}_{Z} & \bar{z}_{Z} \end{bmatrix}^{T}$ is the constant point which instantaneously coincides with \boldsymbol{x}_{Z} , i.e., $\dot{\bar{\boldsymbol{x}}}_{Z} = \boldsymbol{0}$. Then, Eq. (2) is expressed in a dimensionless form as:

$$\frac{\mathrm{d}^2 \tilde{y}}{\mathrm{d}\tilde{t}^2} = \tilde{y}.\tag{24}$$

Eq. (24) is transformed into decoupled linear systems as pointed out in many previous works [18], [19], [20] as



Fig. 5. The development of the LIPM potential along horizontal trajectries, (Initial position and velocity norm is $\begin{bmatrix} x & y & z \end{bmatrix}^{T} = \begin{bmatrix} 0.1 & 0.1 & 0.2 \end{bmatrix}^{T}$ m and 1.2m/s, respectively.)

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{t}} \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix}$$
(25)

$$\begin{bmatrix} \tilde{w}_1\\ \tilde{w}_2 \end{bmatrix} \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1\\ 1 & -1 \end{bmatrix} \begin{bmatrix} \tilde{y}\\ \frac{\mathrm{d}\tilde{y}}{\mathrm{d}\tilde{t}} \end{bmatrix}, \qquad (26)$$

where \tilde{w}_1 and \tilde{w}_2 correspond to normalized unstable and stable modes of the system, respectively, which are abbreviated as DCM (for Divergent Component of Motion) and CCM (for Convergent Component of Motion) in some papers [20]. The combination of them is called the linear inverted pendulum mode (LIPM) [19]. The system is also interpreted as the following scalar potential field:

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{t}} \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} = - \begin{bmatrix} \frac{\partial\phi}{\partial\tilde{w}_1} \\ \frac{\partial\phi}{\partial\tilde{w}_2} \end{bmatrix}, \qquad (27)$$

where

$$\phi(\tilde{w}_1, \tilde{w}_2) \stackrel{\text{def}}{=} -\frac{1}{2} \left(\tilde{w}_1^2 - \tilde{w}_2^2 \right)$$
 (28)

or equivalently

$$\phi(y, \dot{y}, \bar{y}_{\rm Z}) = -\tilde{y} \frac{\mathrm{d}\tilde{y}}{\mathrm{d}\tilde{t}} = -\frac{(y - \bar{y}_{\rm Z})\dot{y}}{\zeta_0 z_0^2},\tag{29}$$

Let us call the above ϕ the LIPM potential. It is enhanced to the 2-dimensional dynamics as

$$\Phi(x, \dot{x}, \bar{x}_{\rm Z}, y, \dot{y}, \bar{y}_{\rm Z}) \stackrel{\rm def}{=} -\frac{(x - \bar{x}_{\rm Z})\dot{x} + (y - \bar{y}_{\rm Z})\dot{y}}{\zeta_0 z_0^2}.$$
 (30)

It is also interpreted as an inner-product $\Phi = -\tilde{r}^{T} \frac{d\tilde{r}}{d\tilde{t}}$. Koolen et al. [21] notified that $\dot{x}(x - \bar{x}_{Z}) < 0$, the lefthand side of which is similar to the above definition is 'a necessary condition for balance'. The LIPM potential does not only make sense in its sign but quantifies the degree of development of the COM motion on an equal energy orbit, which decreases with the development of the COM state. At the zero potential value, the COM motion switches from the motion in which the stable mode is dominant to that in which the unstable mode is. The relation between horizontal motions and the LIPM potential is depicted in Fig. 5. From these facts, it is conceivable that the LIPM



Fig. 6. Stepping strategy using the LIPM potential in a typical case

potential represents the landing urgency. The degree of landing urgency increases as the LIPM potential decreases to a negative value.

The unstable mode or DCM is often referred in order to determine the landing position. The stable mode or CCM is also informative in order to synchronize the foot lifting motion with the COM movement. The LIPM potential involves the both modes, and thus, can be associated with the foot control.

B. Stepping Control of Foot

The author's idea is to design the foot dynamics by referring to the LIPM potential. A typical case of stepping while walking is depicted in Fig. 6. While the LIPM potential decreases from a positive value to zero, the foot can leave the ground and go up. The lifting height should be bounded due to the kinematic limitation. When it decreases under zero, the COM begins to diverge, so that the degree of landing urgency increases and the foot should go down to the ground. One concern is that the ground level is not exactly provided. The idea is to remain a slightly downward speed even around the estimated ground and expect that the foot eventually touches down. The LIPM potential is also informative for the horizontal movement. In the early phase of stepping, the foot should be gradually accelerated for a smooth reorientation toward the desired landing position. As the degree of landing urgency increases, the convergence of the foot motion should be prioritized. An excess stride should also be suppressed. The above discussion suggests that the desired foot motion can be described at the velocity level. To summarize, the referential velocity of the lifting foot ${}^{d}\boldsymbol{v}_{\mathrm{F}} = \begin{bmatrix} {}^{d}\boldsymbol{v}_{x} & {}^{d}\boldsymbol{v}_{y} & {}^{d}\boldsymbol{v}_{z} \end{bmatrix}^{\mathrm{T}}$ should have the following properties:

- 1) ${}^{d}v_{z}$ has the same sign with Φ as long as the foot height is under the nominal upper bound
- 2) ${}^{d}v_{z} < 0$ regardless of Φ , when the foot goes over the nominal upper bound
- 3) ${}^{d}v_{x}$ and ${}^{d}v_{y}$ are proportional to the horizontal error from the destination with respect to the fixed Φ
- 4) $\Phi > 0$ suppresses the gain of ${}^{d}v_{x}$ and ${}^{d}v_{y}$
- 5) $\Phi < 0$ magnifies the gain of ${}^{d}v_{x}$ and ${}^{d}v_{y}$



Fig. 7. Reference velocity of the lifting foot in vertical direction ${}^{\rm d}v_z(z_{\rm F}-h,\Phi)$ ($a=0.1, v_{\rm max}=1$)

- 6) the gain of ${}^{d}v_{x}$ and ${}^{d}v_{y}$ are increased when the foot goes beyond the destination
- 7) ${}^{d}v_{x}$ and ${}^{d}v_{x}$ are bounded

The following design of the referential velocity conforms to the above criteria:

$${}^{d}v_{z}((z_{\rm F}-h),\Phi) = v_{\rm max}(2\alpha(\beta(a(z_{\rm F}-h),\Phi))-1) \quad (31)$$

$${}^{d}v_{x}(x_{\rm F}-{}^{d}x_{\rm F},\Phi) = k_{px}(\alpha(b\Phi)(1-c)+c)({}^{d}x_{\rm F}-x_{\rm F}) \quad (32)$$

$${}^{d}v_{y}(y_{\rm F} - {}^{d}y_{\rm F}, \Phi) = k_{py}(\alpha(b\Phi)(1-c) + c)({}^{d}y_{\rm F} - y_{\rm F})$$
(33)

$$\alpha(x) = \frac{1}{1 + e^{-x}} \tag{34}$$

$$\beta(x,y) = -(x-y) - |x+y|,$$
(35)

where $\boldsymbol{x}_{\rm F} = \begin{bmatrix} x_{\rm F} & y_{\rm F} & z_{\rm F} \end{bmatrix}^{\rm T}$ is the position of the lifting foot, $v_{\rm max}$ is the maximum velocity, k_{px} , k_{py} , a, b and c are parameters to be tuned, and h is the upper bound of the lifting height. The foot is controlled so as to follow ${}^{\rm d}\boldsymbol{v}_{\rm F}$ by

$$\ddot{\boldsymbol{x}}_{\mathrm{F}} = -\boldsymbol{K}_D(\dot{\boldsymbol{x}}_{\mathrm{F}} - {}^{\mathrm{d}}\boldsymbol{v}_{\mathrm{F}}). \tag{36}$$

Eq. (31) is visualized in Fig. 7. In the region where $\Phi > 0$ and $z_{\rm F} - h < 0$, the foot is encouraged to move upward since the landing urgency is low and the foot height is still under the nominal upper bound. On the other hand, when $\Phi < 0$ and $z_{\rm F} - h < 0$, the foot moves downward since the landing urgency is high. When $z_{\rm F} - h > 0$, it prevents lifting foot from further lifting-up over the upper bound. Regarding the horizontal motion, Eq. (32) is visualized in Fig. 8. When $\Phi > 0$, rather a mild movement is preferred since the landing urgency is low. When $\Phi < 0$, the referential velocity increases since it needs to catch up the desired landing position to avoid falling down. The desired landing position is determined by Eq. (18). The pivot foot switches when ZMP moves into the other sole. The oscillation controller defined by Eq. (5) gets the ZMP into the sole of the landed foot, so that the pivot foot is naturally alternated.

IV. SIMULATION

Simulations were conducted in order to validate the proposed stepping controller. The humanoid robot "mighty" [22] was supposed in the simulations. An approximate model in which the total mass concentrates on the COM and the real ZMP coincides with the desired ZMP without delay was assumed for simplicity. The referential joint displacements



Fig. 8. Reference velocity of the lifting foot in horizontal direction ${}^{d}v_x(x_F - {}^{d}x_F, \Phi) \ (b = 10, c = 0.1, k_{px} = 0.1)$

were obtained by solving the inverse kinematics from the desired feet positions and the desired COM position computed from the desired ZMP. The maximum foot height was set for 0.025m and the desired COM height was set for 0.26m. The discrete interval of the simulations was 0.01s.

In the simulation, external forces were applied to the COM in the lateral direction from the supporting foot towards the opposite one. This simulation was conducted to check the robustness of the proposed controller against perturbations during stepping. Fig. 9 shows some snapshots of the simulation from 2.50s to 2.90s. The loci of the feet, the COM and the ZMP are drawn in Fig. 10. It is seen that the robot achieved alternate stepping successfully regardless of the exertion of the external forces. The robot stepped out naturally when perturbed and succeeded to avoid falling.

For comparison, the resulted loci of the COM, the ZMP and feet with the previous method [10] were plotted in Fig. 11. The stepping foot landed soon after the perturbation with rather a small step width with the previous method. When the perturbation was applied 0.01s longer, the robot controlled by the previous method failed to absorb the perturbation and fell down, while it successfully avoided falling when controlled by the proposed method. It typically presents the efficacy of the proposed controller over the previous behavior. Though it is possible to improve the lateral response of the foot by increasing the horizontal gain and decreasing the vertical gain, such a choice may degrade the synchronicity of the foot to the COM movement. Namely, the movement obtained by the proposed controller is hardly emerged by the previous method even with finely tuned gains.

V. CONCLUSIONS

A novel foot stepping controller for biped robots was proposed. The LIPM potential was newly defined as the measure of the degree of development of the COM motion and that of landing urgency. It suggests criteria of a preferable foot movement at the velocity level. Stable alternating stepping motion was achieved by combining with the COM-ZMP oscillator, and flexible step-out motions were naturally emerged against perturbations. The vertical referential velocity is designed to remain downward in order to land on unknown ground level. A shock absorption and terrain-adapting control is additionally required in practice, which is a future work. While a basic stepping control is addressed in this paper, the authors believe that the proposed method supports the fundamental stabilization and mobility



Fig. 9. Snapshots of a simulation of an alternating stepping control during which perturbations were applied



Fig. 10. Loci of the COM, the ZMP and feet when external forces applied with the proposed method



Fig. 11. Loci of the COM, the ZMP and feet when external forces applied with the previous method [10]

of the robot, on which all abilities to execute higher tasks stand.

ACKNOWLEDGMENT

The authors would like to express a cordial appreciation to Mr. Hiroshi Atsuta for his technical support to this work.

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